Electromagnetic interactions of particles with matter

November 4, 2002

Abstract

This document is a brief review to the main mechanisms of electromagnetic interactions of charged particles and photons with matter, pertinent in Particle Physics, and their inplementation in GEANT4.

’Standard’ em physics : the model

The projectile is assumed to have an energy $\geq$ 1 keV.

- The atomic electrons are quasi-free : their binding energy is neglected (except for photoelectric effect).
- The atomic nucleus is fixe : the recoil momentum is neglected.

The matter is described as homogeneous, isotropic, amorphous.

1. Common to all charged particles
   - ionization ($\sim keV \longrightarrow$)
   - Coulomb scattering from nuclei ($\sim keV \longrightarrow$)
   - Cerenkov effect
   - Scintillation
   - transition radiation
2. Muons
   - (e+,e-) pair production ($\sim 100GeV \longrightarrow$)
   - bremsstrahlung ($\sim 100GeV \longrightarrow$)
   - nuclear interaction ($\sim 1TeV \longrightarrow$)
3. Electrons and positrons
   - bremsstrahlung ($\sim 10MeV \longrightarrow$)
   - e+ annihilation

4. Photons
   - gamma conversion ($\sim 10MeV \longrightarrow$)
   - incoherent scattering ($\sim 100keV \longrightarrow \sim 10MeV$)
   - photo electric effect ($\longleftarrow \sim 100keV$)
   - coherent scattering ($\longleftarrow \sim 100keV$)
5. Optical photons
   - reflection and refraction
   - absorption
   - Rayleigh scattering

Total :$\sim$ 15 processes $\longrightarrow$ $\sim$ 40 classes

+ $\sim$ 10 classes for the materials category

A few words about the GEANT4 processes in general

A process may have three types of actions :

- well located in space : PostStep action
- not well located in space : AlongStep action
- well located in time : AtRest action

Each action is twofold :

- predicts where/when the interaction will occur : `GetPhysicalInteractionLength()`
- computes the final state of the interaction, where/when it occurs : `DoIt()`

A process has to fill 1, 2 or 3 couples of the following methods :

	AtRest	AlongStep	PostStep
GetPhysicalInteractionLength()			
DoIt()			

- DiscreteProcess is shortcut for a process which have **only** PostStep action.
- ContinuousProcess is shortcut for a process which have **only** AlongStep action.
- AtRestProcess is shortcut for a process which have **only** AtRest action.

examples

- discrete process : Compton scattering
  step determined by cross section, interaction at the end of the step (PostStepAction).
- continuous process : Cerenkov effect
  photons are created along the step, nb of photons (roughly) proportional to the step length (AlongStepAction).
- at rest process : no displacement, time is the relevant variable, e.g. positron annihilation at rest.

These are the 'pure' process types.

Some of the e.m. processes have combinations of actions :

- ionisation : continuous (energy loss) + discrete (Moller/Bhabha scattering, knock-on electron production)
- bremsstrahlung : continuous (energy loss due to soft photons) + discrete (hard photon emission)

in both cases the production threshold separates the continuous and discrete part of the process :

- if the (kinetic) energy of the secondary $\leq$ threshold energy, the secondary is not created , the effect of these soft interactions are treated as a continuous energy loss
- if the energy of the secondary is big enough, it is created at the end of the step (discrete part)

PhysicsList

For each type of particle the ProcessManager maintains a list of processes to be apply.

More precisely, there are 3 ordered lists of processes :

- AtRest action
- AlongStep action
- PostStep action

These lists are registered in the `UserPhysicsList` class.

example of PhysicsList

```
if (particleName == "e-") {
pmanager->AddProcess(new G4MultipleScattering, -1, 1,1);
pmanager->AddProcess(new G4eIonisation,        -1, 2,2);
pmanager->AddProcess(new G4eBremsstrahlung,    -1,-1,3);
}

else if (particleName == "e+") {
pmanager->AddProcess(new G4MultipleScattering, -1, 1,1);
pmanager->AddProcess(new G4eIonisation,        -1, 2,2);
pmanager->AddProcess(new G4eBremsstrahlung,    -1,-1,3);
pmanager->AddProcess(new G4eplusAnnihilation,   0,-1,4);
}
```

```
if (particleName == "mu+" || particleName == "mu-") {
pmanager->AddProcess(new G4MultipleScattering, -1, 1,1);
pmanager->AddProcess(new G4MuIonisation,        -1, 2,2);
pmanager->AddProcess(new G4MuBremsstrahlung,    -1,-1,3);
pmanager->AddProcess(new G4MuPairProduction,    -1,-1,4);
}

if ((particle->GetPDGCharge() != 0.0) &&
(!particle->IsShortLived()) &&
(particle->GetParticleName() != "chargedgeantino")) {
pmanager->AddProcess(new G4MultipleScattering, -1,1,1);
pmanager->AddProcess(new G4hIonisation,         -1,2,2);
}
```

```
if (particleName == "gamma") {
pmanager->AddDiscreteProcess(new G4PhotoElectricEffect);
pmanager->AddDiscreteProcess(new G4ComptonScattering);
pmanager->AddDiscreteProcess(new G4GammaConversion);
}
```

is a shortcut for :

```
pmanager->AddProcess(new G4PhotoElectricEffect, -1,-1,1);
pmanager->AddProcess(new G4ComptonScattering,   -1,-1,2);
pmanager->AddProcess(new G4GammaConversion,     -1,-1,3);
```

For processes which have only PostStepAction, the ordering is not important.

Compton scattering

The Compton effect describes the scattering off quasi-free atomic electrons :

$$\gamma + e \rightarrow \gamma' + e'$$

Each atomic electron acts as an independent cible; Compton effect is called incoherent scattering. Thus:

$$\text{cross section per atom} = Z \times \text{cross section per electron}$$

The inverse Compton scattering also exists: an energetic electron collides with a low energy photon which is blue-shifted to higher energy. This process is of importance in astrophysics.

Compton scattering is related to (e^+, e^-) annihilation by crossing symmetry.

energy spectrum

Under the same assumption,the unpolarized differential cross section per atom is given by the Klein-Nishina formula [Klein29] :

$$\frac{d\sigma}{dk'} = \frac{\pi\, r_e^2}{mc^2} \frac{Z}{\kappa^2} \left[\epsilon + \frac{1}{\epsilon} - \frac{2}{\kappa} \left(\frac{1-\epsilon}{\epsilon} \right) + \frac{1}{\kappa^2} \left(\frac{1-\epsilon}{\epsilon} \right)^2 \right] \qquad (1)$$

where

- k' energy of the scattered photon ; $\epsilon = k'/k$
- r_e classical electron radius
- κ k/mc^2

total cross section per atom

$$\sigma(k) = \int_{k'_{min}=k/(2\kappa+1)}^{k'_{max}=k} \frac{d\sigma}{dk'}\, dk'$$

$$\sigma(k) = 2\pi\, r_e^2\, Z \left[\left(\frac{\kappa^2 - 2\kappa - 2}{2\kappa^3} \right) \ln(2\kappa+1) + \frac{\kappa^3 + 9\kappa^2 + 8\kappa + 2}{4\kappa^4 + 4\kappa^3 + \kappa^2} \right]$$

limits

$k \to \infty :$ σ goes to 0 : $\sigma(k) \sim \pi\, r_e^2\, Z\, \frac{\ln 2\kappa}{\kappa}$

$k \to 0 :$ $\sigma \to \frac{8\pi}{3}\, r_e^2\, Z$ (classical Thomson cross section)

low energy limit

In fact, when $k \leq 100\ keV$ the binding energy of the atomic electron must be taken into account by a corrective factor to the Klein-Nishina cross section:

$$\frac{d\sigma}{dk'} = \left[\frac{d\sigma}{dk'}\right]_{KN} \times S(k,k')$$

See for instance [Cullen97] or [Salvat96] for derivation(s) and discussion of the *scattering function* S(k,k').

As a consequence, at very low energy, the total cross section goes to 0 like k^2. It also suppresses the forward scattering.

At X-rays energies the scattering function has little effect on the Klein-Nishina energy spectrum formula 1. In addition the Compton scattering is not the dominant process in this energy region.

total cross section per atom in GEANT4

The total cross section has been parametrized :

$$\sigma(Z,\kappa) = \left[P_1(Z)\,\frac{\log(1+2\kappa)}{\kappa} + \frac{P_2(Z) + P_3(Z)\kappa + P_4(Z)\kappa^2}{1 + a\kappa + b\kappa^2 + c\kappa^3} \right]$$

where:

$$\begin{aligned} \kappa &= k/mc^2 \\ P_i(Z) &= Z(d_i + e_i Z + f_i Z^2) \end{aligned}$$

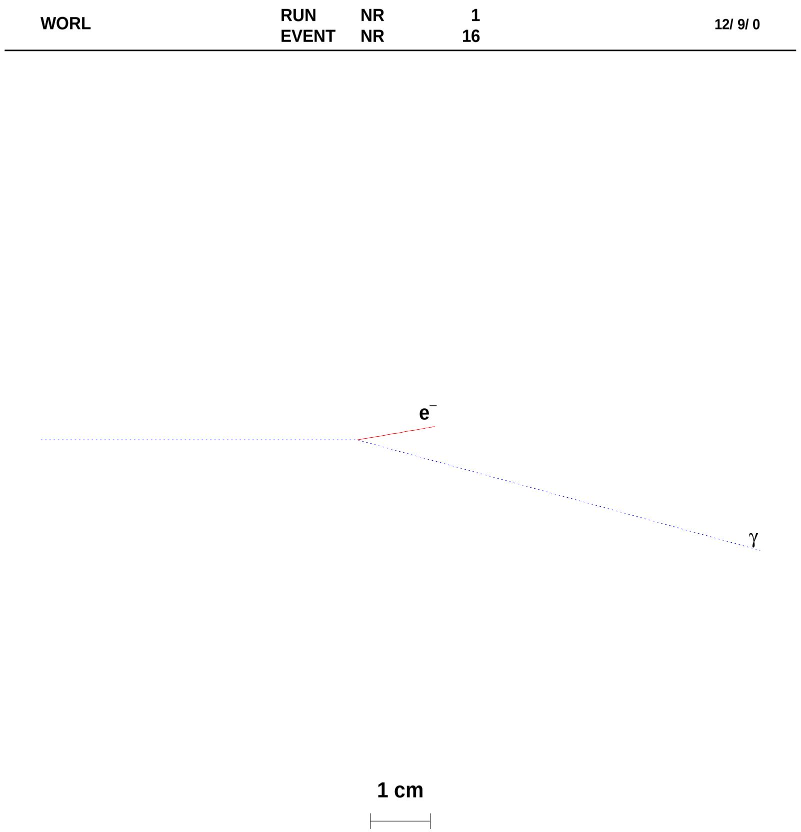
The fit was made over 511 data points chosen between:

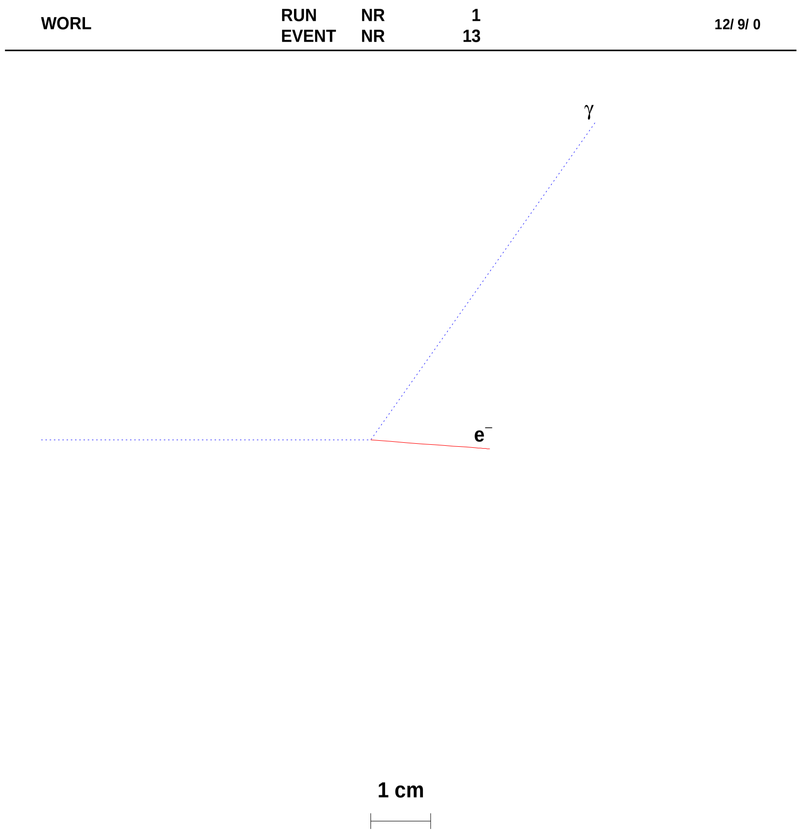
$$1 \le Z \le 100 \qquad ; \qquad k \in [10\ \text{keV}, 100\ \text{GeV}]$$

The accuracy of the fit is estimated to be:

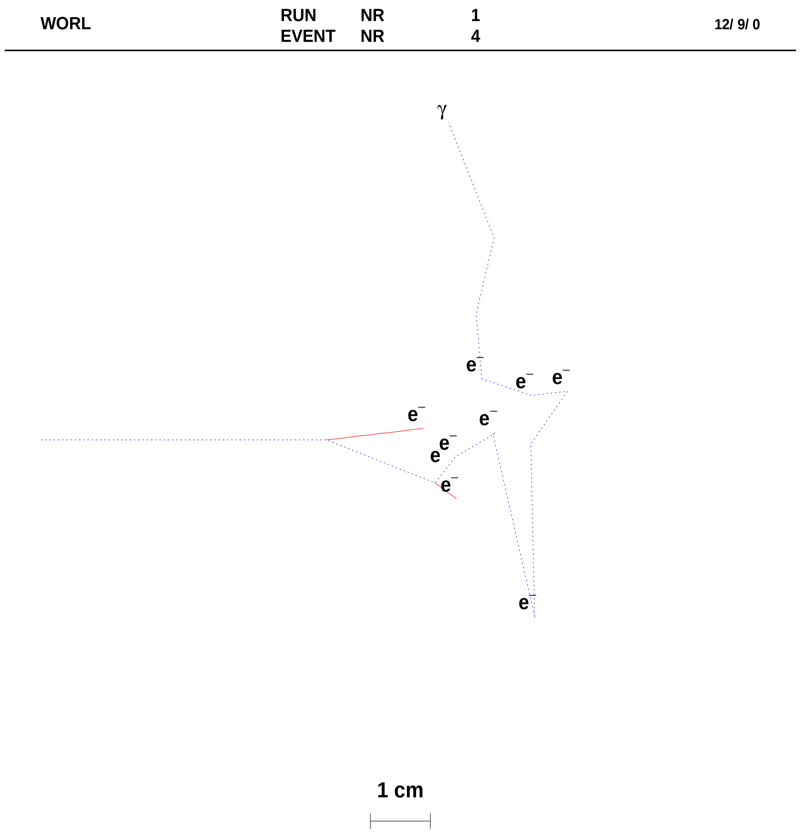
$$\frac{\Delta\sigma}{\sigma} = \begin{cases} \approx 10\% & \text{for } k \simeq 10\ \text{keV} - 20\ \text{keV} \\ \le 5 - 6\% & \text{for } k > 20\ \text{keV} \end{cases}$$

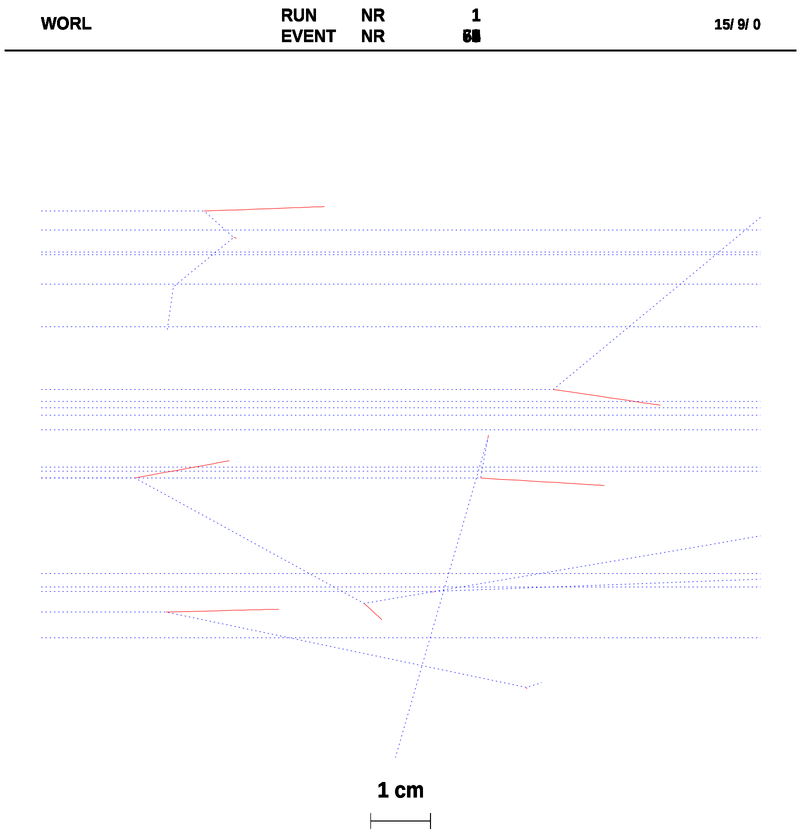
γ 10 MeV in 10 cm Aluminium: Compton scattering





γ 10 MeV in 10 cm Aluminium: Compton scattering





Gamma conversion in (e^+, e^-) pair

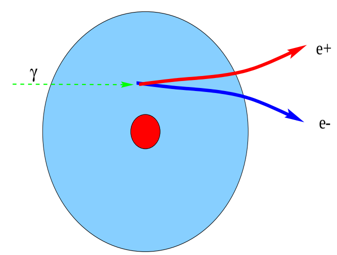
This is the transformation of a photon into an (e^+, e^-) pair in the Coulomb field of atoms (for momentum conservation).

To create the pair, the photon must have at least an energy of $2mc^2(1 + m/M_{rec})$.

Theoretically, (e^+, e^-) pair production is related to bremsstrahlung by crossing symmetry:

- incoming e^- $\leftrightarrow$ outgoing e^+
- outgoing $\gamma \leftrightarrow$ incoming γ

For $E_\gamma \geq$ few tens MeV, (e^+, e^-) pair creation is the dominant process for the photon, in all materials.



differential cross section

The differential cross section is given by the Bethe-Heitler formula [Heitl57], corrected and extended for various effects:

- the screening of the field of the nucleus
- the pair creation in the field of atomic electrons
- the correction to the Born approximation
- the LPM suppression mechanism
- ...

See Seltzer and Berger for a synthesis of the theories [Sel85].

high energies regime : $E_\gamma \gg m_e c^2/(\alpha Z^{1/3})$

Above few GeV the energy spectrum formula becomes simple :

$$\left.\frac{d\sigma}{d\epsilon}\right]_{Tsai} \approx 4\alpha\, r_e^2 \times \left\{\left[1-\frac{4}{3}\epsilon(1-\epsilon)\right]\left(Z^2\left[L_{rad}-f(Z)\right]+Z L'_{rad}\right)\right\} \qquad (2)$$

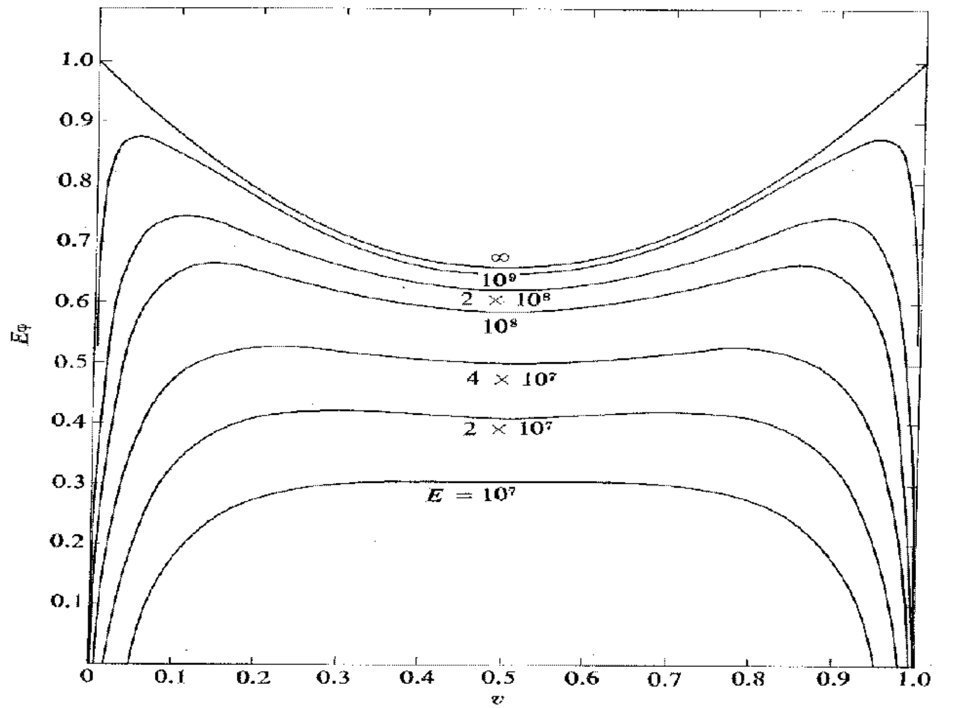
where

E_γ	energy of the incident photon
E	total energy of the created e^+ (or e^-) ; $\epsilon = E/E_\gamma$
$L_{rad}(Z)$	$\ln(184.15/Z^{1/3})$ (for $Z \geq 5$)
$L'_{rad}(Z)$	$\ln(1194/Z^{2/3})$ (for $Z \geq 5$)
$f(Z)$	Coulomb correction function

energy spectrum

limits: $E_{min} = mc^2$: no infrared divergence. $E_{max} = E_\gamma - mc^2$.

The partition of the photon energy between e^+ and e^- is flat at low energy ($E_\gamma \leq 50\ MeV$) and increasingly asymmetric with energy. For $E_\gamma > TeV$ the LPM effect reinforces the asymmetry.



total cross section per atom in GEANT4

E_γ = incident gamma energy, and $X = \ln(E_\gamma / m_e c^2)$

The total cross-section has been parameterised as :

$$\sigma(Z, E_\gamma) = Z(Z+1)\left[F_1(X) + F_2(X)\,Z + \frac{F_3(X)}{Z}\right]$$

with :

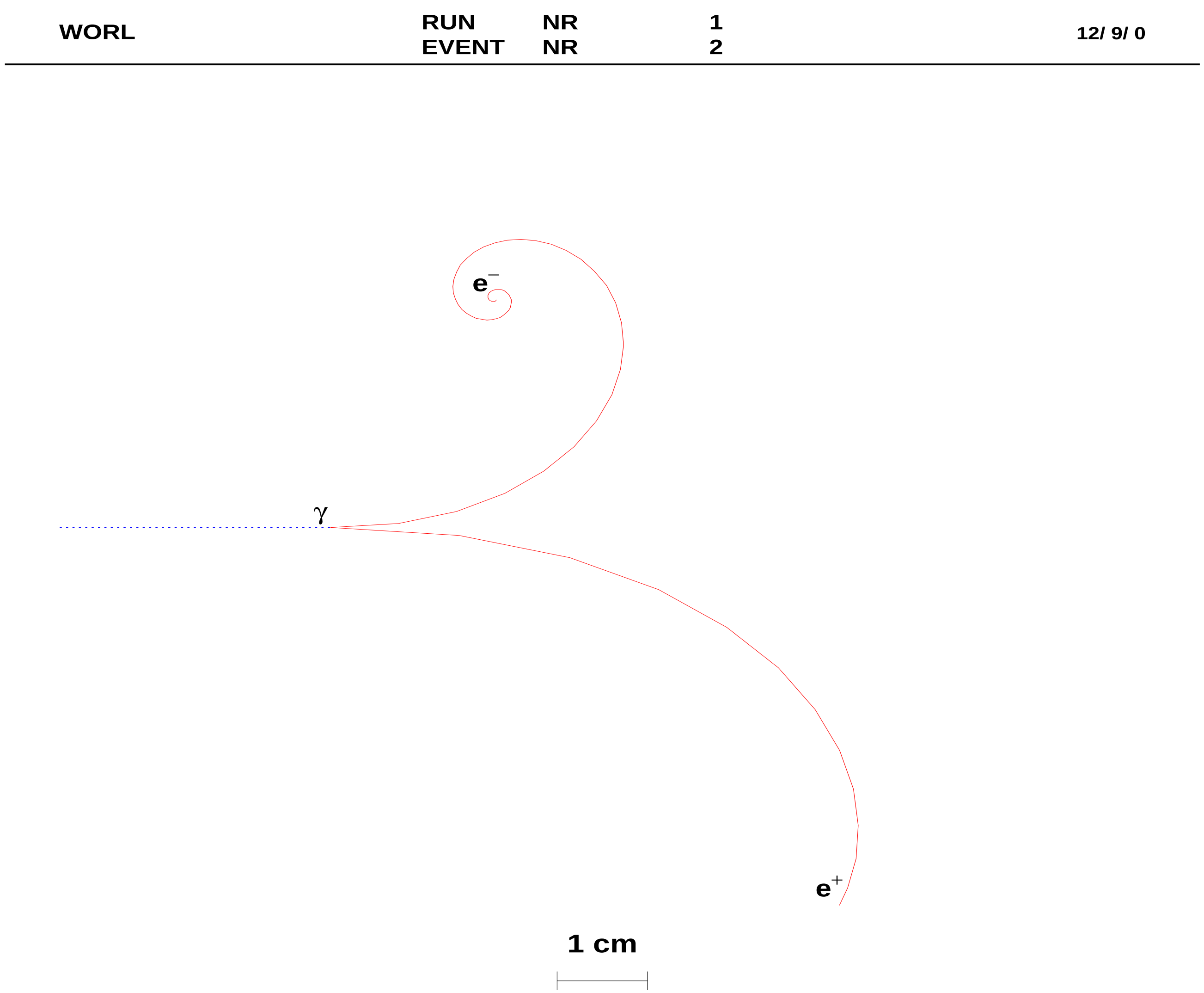
$$\begin{aligned}
F_1(X) &= a_0 + a_1 X + a_2 X^2 + a_3 X^3 + a_4 X^4 + a_5 X^5 \\
F_2(X) &= b_0 + b_1 X + b_2 X^2 + b_3 X^3 + b_4 X^4 + b_5 X^5 \\
F_3(X) &= c_0 + c_1 X + c_2 X^2 + c_3 X^3 + c_4 X^4 + c_5 X^5
\end{aligned}$$

The parameters a_i, b_i, c_i were fitted to the data [hubb80].

This parameterisation describes the data in the range :

$$\left.\begin{array}{c} 1 \leq Z \leq 100 \\ E_\gamma \in [1.5\ \text{MeV}, 100\ \text{GeV}] \end{array}\right\} \frac{\Delta\,\sigma}{\sigma} \leq 5\% \quad \text{with a mean value of} \approx 2.2\%$$

γ 200 MeV in 10 cm Aluminium. Field 5 tesla



Ionization

The basic mechanism is an inelastic collision of the moving charged particle with the atomic electrons of the material, ejecting off an electron from the atom :

$$\mu + atom \rightarrow \mu + atom^{+} + e^{-}$$

In each individual collision, the energy transfered to the electron is small. But the total number of collisions is large, and we can well define the average energy loss per (macroscopic) unit path length.

Mean energy loss and energetic δ-rays

$$\frac{d\sigma(Z,E,T)}{dT}$$

is the differential cross-section per atom for the ejection of an electron with kinetic energy T by an incident charged particle of total energy E moving in a material of density ρ.

One may wish to take into account separately the high-energy knock-on electrons produced above a given threshold T_{cut} (miss detection, explicit simulation ...).

$T_{cut} \gg I$ (mean excitation energy in the material).

$T_{cut} > 1$ keV in Geant4

Below this threshold, the soft knock-on electrons are counted only as continuous energy lost by the incident particle.

Above it, they are explicitly generated. Those electrons must be excluded from the mean continuous energy loss count.

The mean rate of the energy lost by the incident particle due to the soft δ-rays is :

$$\frac{dE_{soft}(E,T_{cut})}{dx} = n_{at} \cdot \int_0^{T_{cut}} \frac{d\sigma(Z,E,T)}{dT} T\, dT \tag{3}$$

n_{at} : nb of atoms per volume in the matter.

The total cross-section per atom for the ejection of an electron of energy $T > T_{cut}$ is :

$$\sigma(Z,E,T_{cut}) = \int_{T_{cut}}^{T_{max}} \frac{d\sigma(Z,E,T)}{dT}\, dT \tag{4}$$

where T_{max} is the maximum energy transferable to the free electron.

Mean rate of energy loss by heavy particles

The integration of 3 leads to the well known Bethe-Bloch truncated energy loss formula [PDG] :

$$\left.\frac{dE}{dx}\right]_{T<T_{cut}} = 2\pi r_e^2 mc^2 n_{el} \frac{(z_p)^2}{\beta^2} \times \left[\ln\left(\frac{2mc^2\beta^2\gamma^2 T_{up}}{I^2}\right) - \beta^2\left(1 + \frac{T_{up}}{T_{max}}\right) - \delta - \frac{2C_e}{Z}\right]$$

Fluctuations in energy loss

$\langle \Delta E \rangle = (dE/dx).\Delta x$ gives only the average energy loss by ionization. There are fluctuations. Depending of the amount of matter in Δx the distribution of ΔE can be strongly asymmetric ($\rightarrow$ the Landau tail).

The large fluctuations are due to a small number of collisions with large energy transfers.

Energy loss fluctuations : the model in GEANT

Based on a very simple model of the particle-atom interaction.

The atoms are assumed to have only two energy levels E_1 and E_2.

The particle-atom interaction can be :

- an excitation with energy loss E_1 or E_2
- an ionization with energy loss distribution $g(E) \sim 1/E^2$.

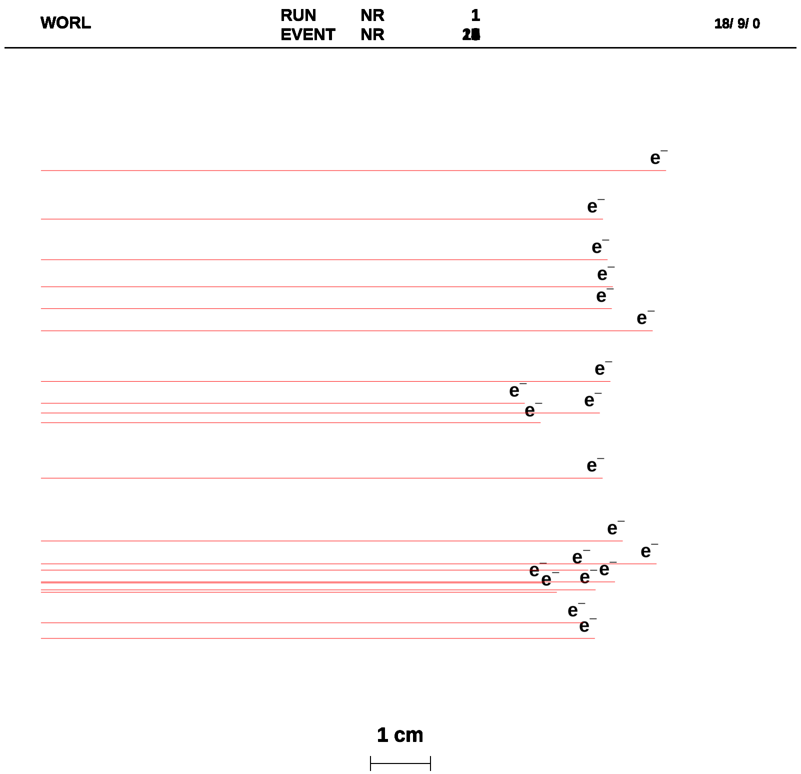
This simple model of the energy loss fluctuations is rather fast and it can be used for any thickness of the materials, and for any T_{cut}.

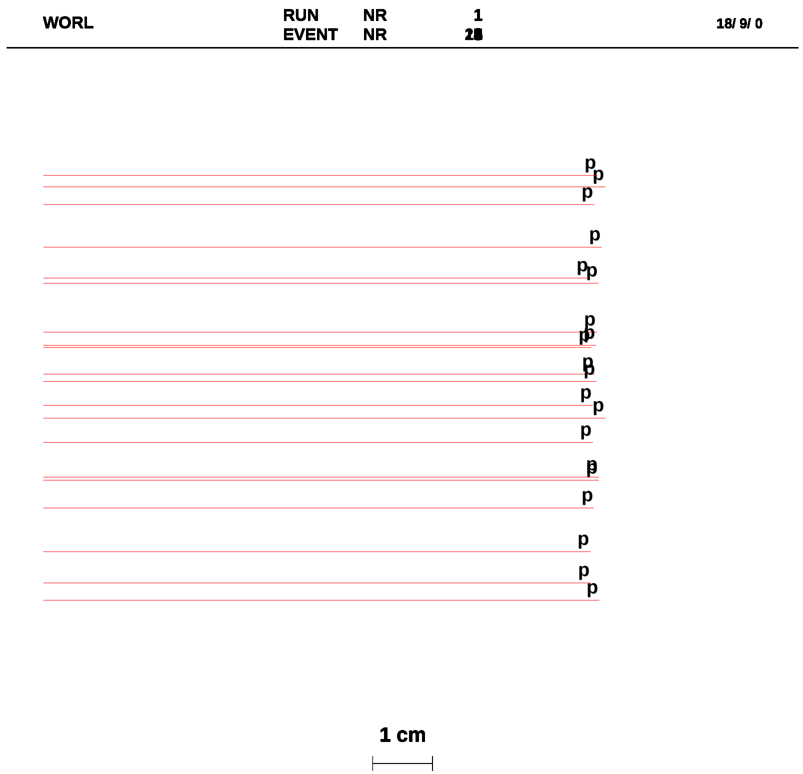
This has been proved performing many simulations and comparing the results with experimental data, see e.g [Urban95].

Approaching the limit of the validity of Landau's theory, the loss distribution approaches smoothly the Landau form.

Fluctuations on ΔE lead to fluctuations on the actual range (straggling).

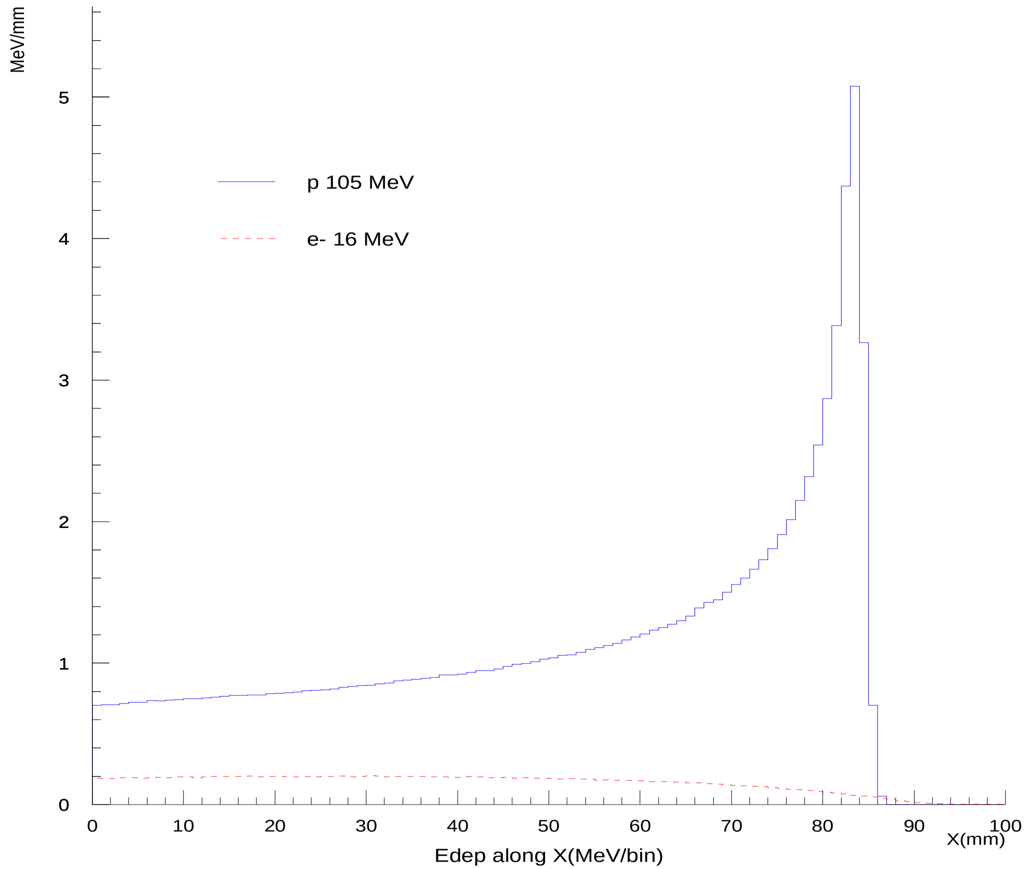
penetration of e^- (16 MeV) and proton (105 MeV) in 10 cm of water.





Bragg curve. More energy per unit length are deposit towards the end of trajectory rather at its beginning.

proton 105 MeV in water: Bragg peak



Energetic δ rays

The differential cross-section per atom for producing an electron of kinetic energy T, with $I \ll T_{cut} \leq T \leq T_{max}$, can be written :

$$\frac{d\sigma}{dT} = 2\pi r_e^2 mc^2 Z \frac{z_p^2}{\beta^2} \frac{1}{T^2} \left[1 - \beta^2 \frac{T}{T_{max}} + \frac{T^2}{2E^2} \right]$$

(the last term for spin 1/2 only).

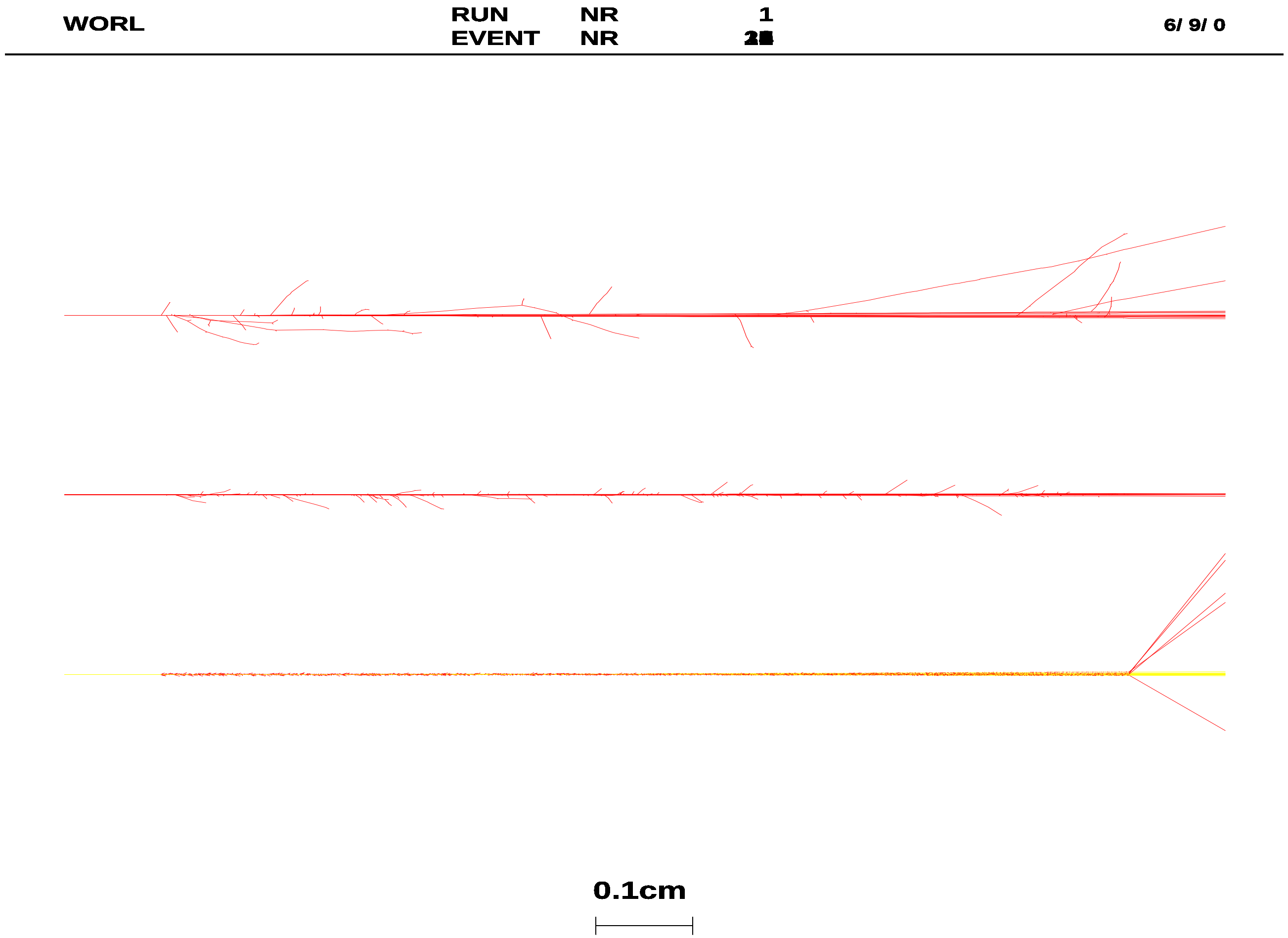
The integration (4) gives :

$$\sigma(Z, E, T_{cut}) = \frac{2\pi r_e^2 Z z_p^2}{\beta^2} \left[\left(\frac{1}{T_{cut}} - \frac{1}{T_{max}} \right) - \frac{\beta^2}{T_{max}} \ln \frac{T_{max}}{T_{cut}} + \frac{T_{max} - T_{cut}}{2E^2} \right]$$

(the last term for spin 1/2 only).

delta rays

200 MeV electrons, protons, alphas in 1 cm of Aluminium



Incident electrons and positrons

For incident $e^{-/+}$ the Bethe Bloch formula must be modified because of the mass and identity of particles (for e^{-}).

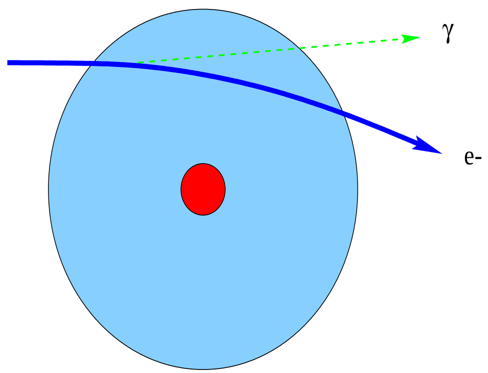
One use the Moller or Bhabha cross sections [Mess70] and the Berger-Seltzer dE/dx formula [ICRU84, Selt84].

Bremsstrahlung

A fast moving charged particle is decelerated in the Coulomb field of atoms. A fraction of its kinetic energy is emitted in form of real photons.

The probability of this process is $\propto 1/M^2$ (M: masse of the particle) and $\propto Z^2$ (atomic number of the matter).

Above a few tens MeV, bremsstrahlung is the dominant process for e- and e+ in most materials. It becomes important for muons (and pions) at few hundred GeV.



differential cross section

The differential cross section is given by the Bethe-Heitler formula [Heitl57], corrected and extended for various effects:

- the screening of the field of the nucleus
- the contribution to the brems from the atomic electrons
- the correction to the Born approximation
- the polarisation of the matter (dielectric suppression)
- the so-called LPM suppression mechanism
- ...

See Seltzer and Berger for a synthesis of the theories [Sel85].

Energetic photons and truncated energy loss rate

One may wish to take into account separately the high-energy photons emitted above a given threshold k_{cut} (miss detection, explicit simulation ...).

Those photons must be excluded from the mean energy loss count.

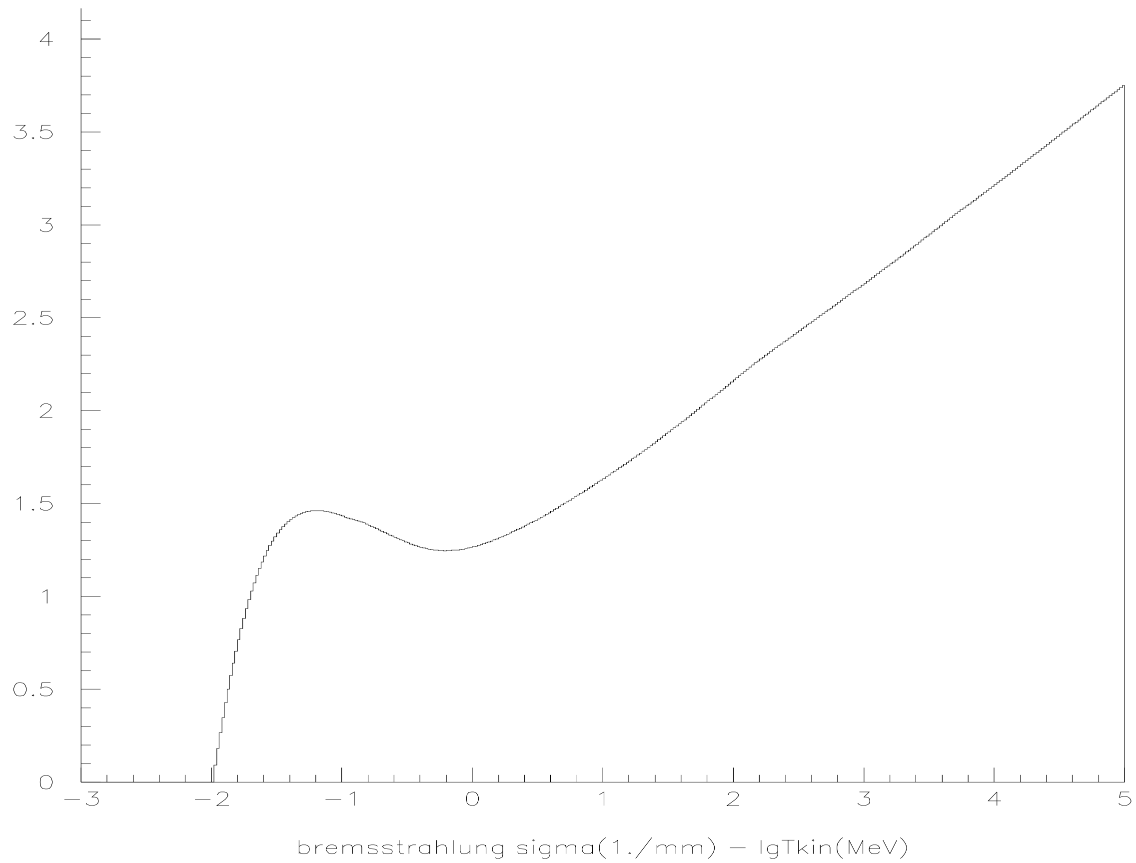
$$-\frac{dE}{dx}\bigg]_{k<k_{cut}} = n_{at} \int_{k_{min}=0}^{k_{cut}} k \, \frac{d\sigma}{dk} \, dk \tag{5}$$

n_{at} is the number of atoms per volume.

Then, the truncated total cross-section for emitting 'hard' photons is:

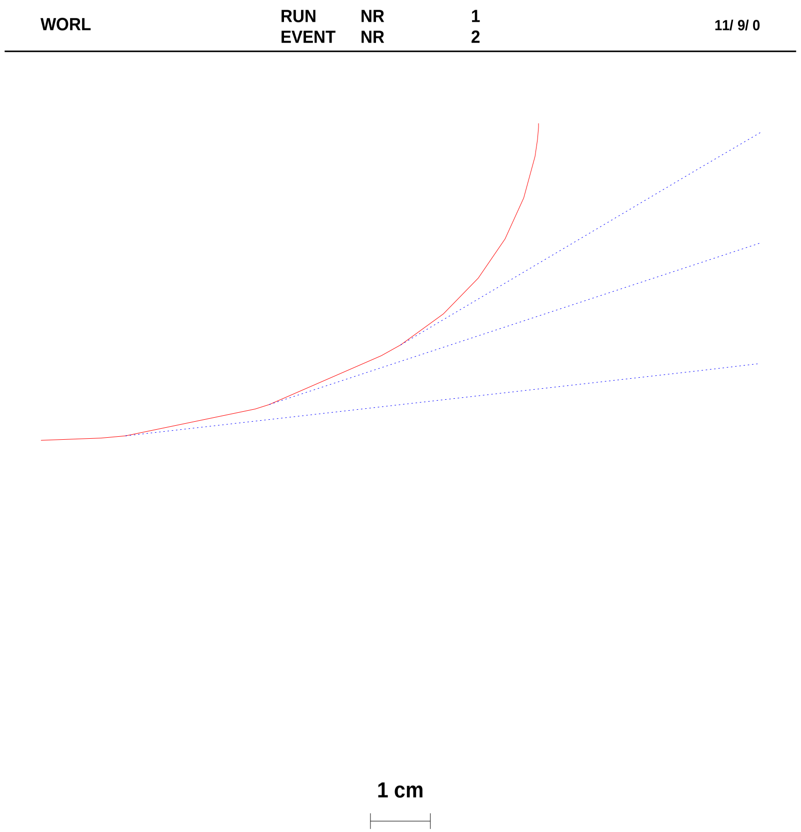
$$\sigma(E, k_{cut} \leq k \leq k_{max}) = \int_{k_{cut}}^{k_{max} \approx E} \frac{d\sigma}{dk} \, dk \tag{6}$$

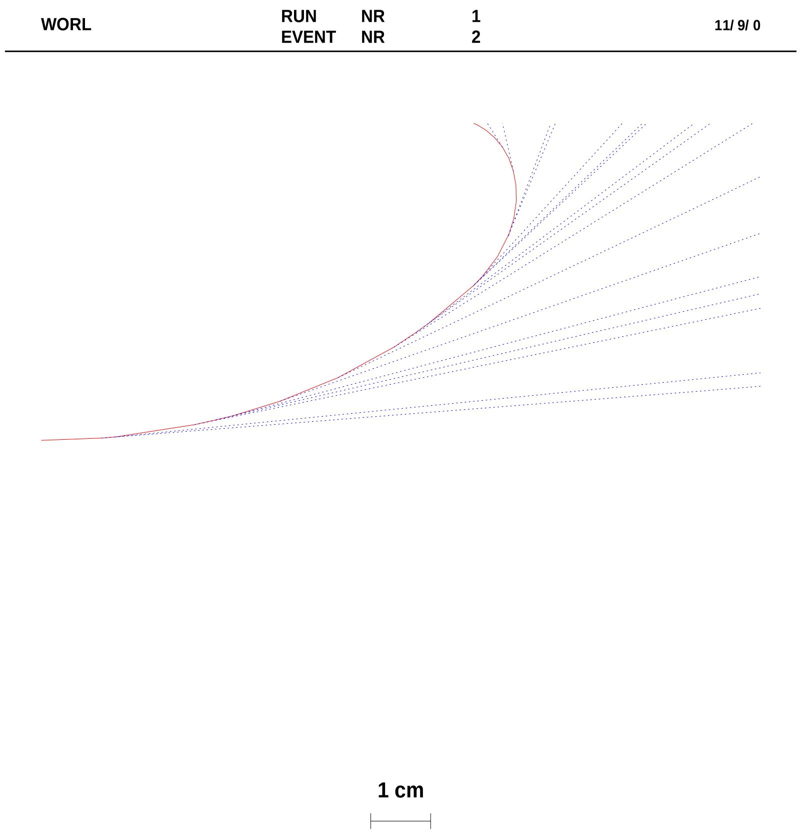
number of interactions per mm in Lead (cut 10 keV)



electron kinetic energy (ln(MeV))

e^- 200 MeV in 10 cm Aluminium (cut: 1 MeV, 10 keV). Field 5 tesla





formation length ([Antho96])

In the bremsstrahlung process the longitudinal momentum transfer from the nucleus to the electron can be very small. For $E \gg mc^2$ and $E \gg k$:

$$q_{long} \sim \frac{k(mc^2)^2}{2E(E-k)} \sim \frac{k}{2\gamma^2}$$

Thus, the uncertainty principle requires that the emission take place over a comparatively long distance :

$$f_v \sim \frac{2\hbar c\gamma^2}{k} \tag{7}$$

f_v is called the formation length for bremsstrahlung in vacuum.
It is the distance of coherence, or the distance required for the electron and photon to separate enough to be considered as separate particles. If anything happens to the electron or photon while traversing this distance, the emission can be disrupted.

Landau-Pomeranchuk-Migdal suppression mechanism

The electron can multiple scatter with the atoms of the medium while it is still in the formation zone. If the angle of multiple scattering, θ_{ms}, is greater than the typical emission angle of the emitted photon, $\theta_{br} = mc^2/E$, the emission is suppressed.

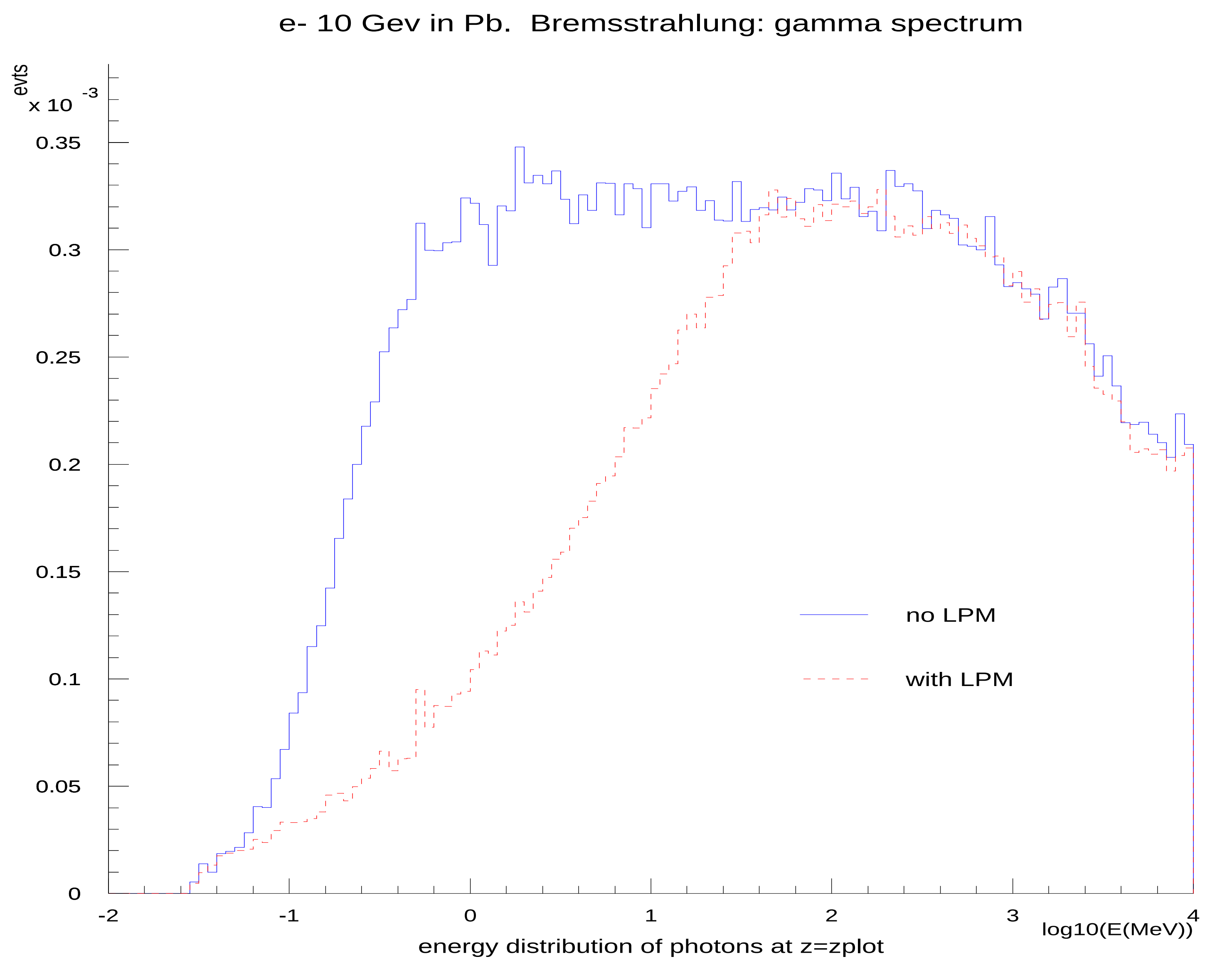
In the gaussian approximation : $\theta_{ms}^2 = \frac{2\pi}{\alpha}\frac{1}{\gamma^2}\frac{f_v(k)}{X_0}$ where f_v is the formation length in vacuum, defined in equation 7.

Writing $\theta_{ms}^2 > \theta_{br}^2$ show that suppression becomes signifiant for photon energies below a certain value, given by

$$\frac{k}{E} < \frac{E}{E_{lpm}} \tag{8}$$

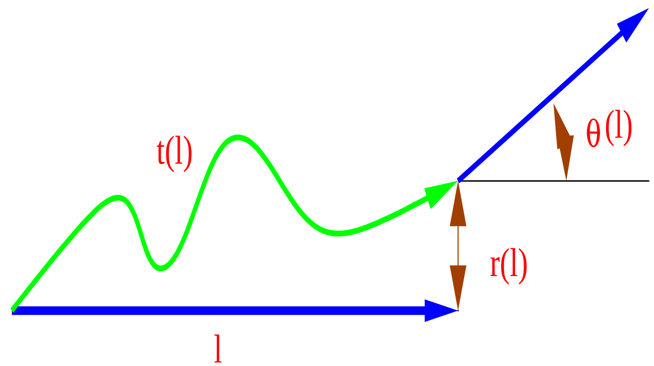
E_{lpm} is a characteristic energy of the effect :

$$E_{lpm} = \frac{\alpha^2}{4\pi}\frac{mc^2}{r_e}\,X_0 \;\sim (7.7\,TeV/cm) \times X_0\,(cm) \tag{9}$$


e- 10 Gev in Pb. Bremsstrahlung: gamma spectrum
evts
x 10 -3
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-2
-1
0
1
2
3
4
log10(E(MeV))
energy distribution of photons at z=zplot
no LPM
with LPM

Multiple Coulomb scattering

Charged particles traversing a finite thickness of matter suffer repeated elastic Coulomb scattering. The cumulative effect of these small angle scatterings is a net deflection from the original particle direction.



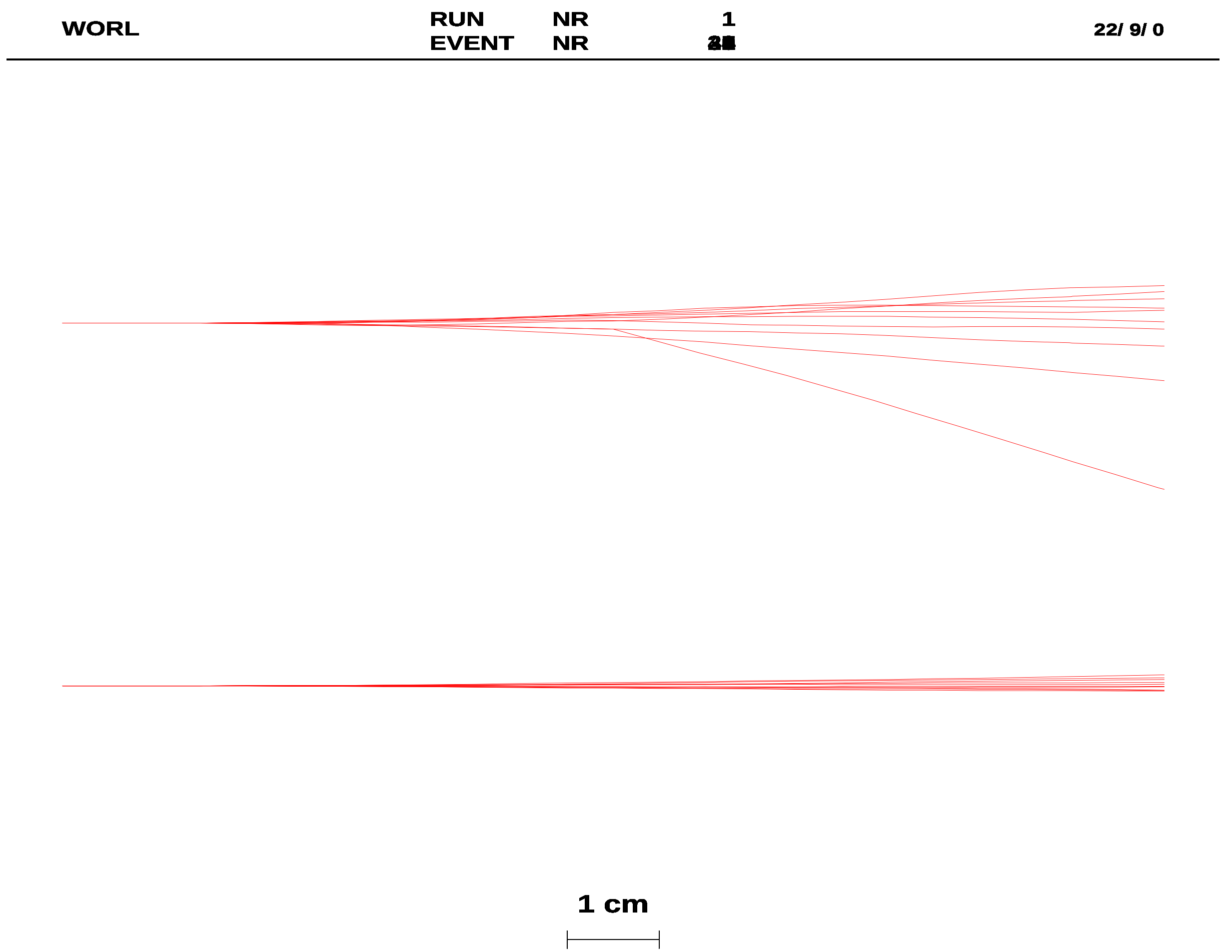
If the number of individual collisions is enough (> 20) the multiple Coulomb scattering angular distribution is gaussian at small angles and like Rutherford scattering at large angles.

The Molière theory reproduces rather well this distribution.

[Mol48, Bethe53]

Energy dependence

10 π^+ of 200 MeV and 1 GeV crossing 10 cm of Aluminium.



Others models for simulation

Several models of multiple Coulomb scattering simulation algorithms have been proposed, not necessarily based on the Molière theory. (See the references in [PDG00])
For instance :

- J.M. Fernandez-Verea et al. : a "mixed" (detailed + condensed) model. [Fer93]
- L.Urban : a condensed model based on Lewis theory.[Urb00]

backscattering of low energy electrons

not a trivial task for a code designed for high energy physics applications in mind

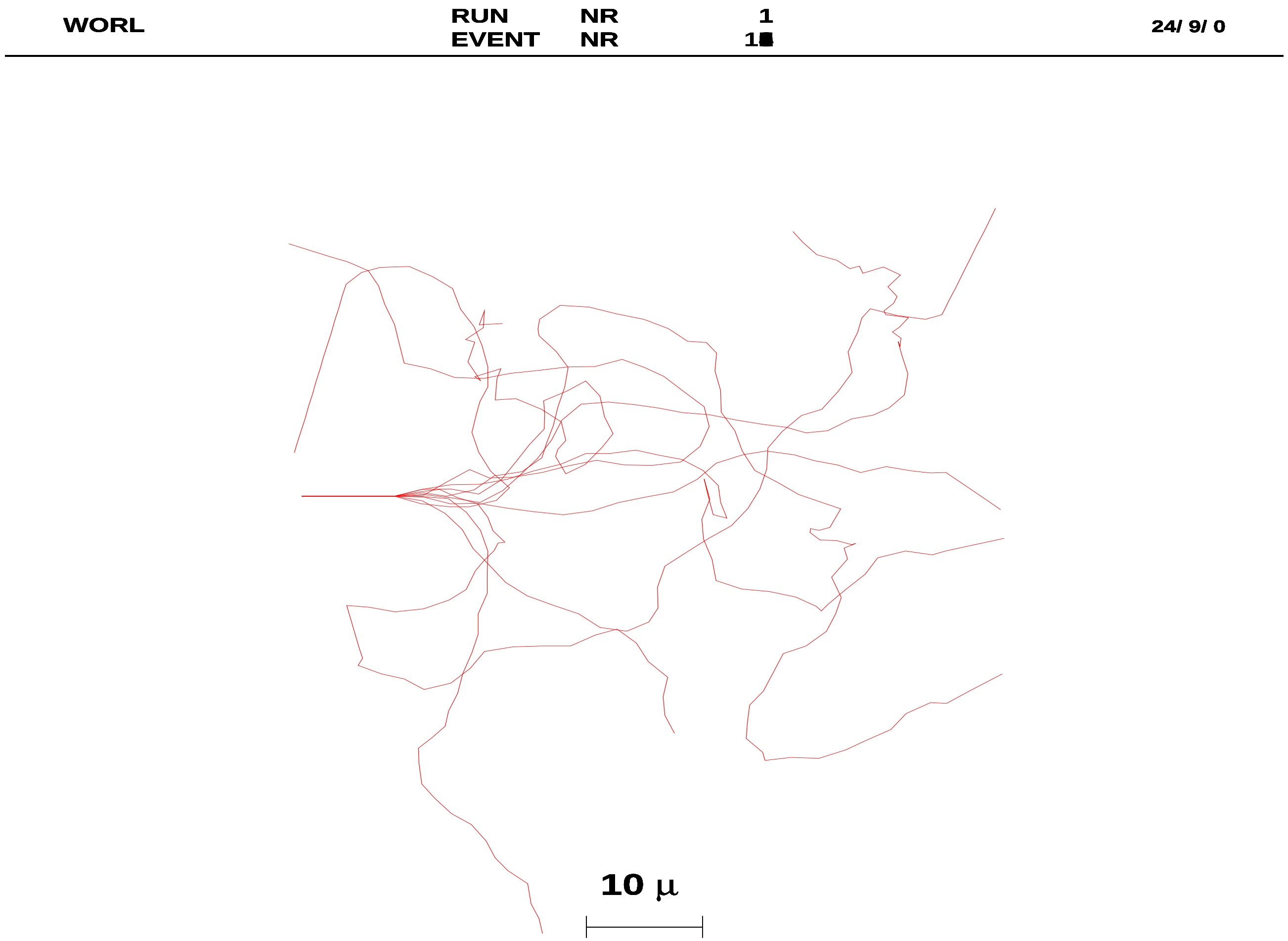
e.g. GEANT3 is not able to reproduce the experimental data, - sim.results are far from data and they are unstable, a small change in the cut or max.step limitation triggers big changes in the results

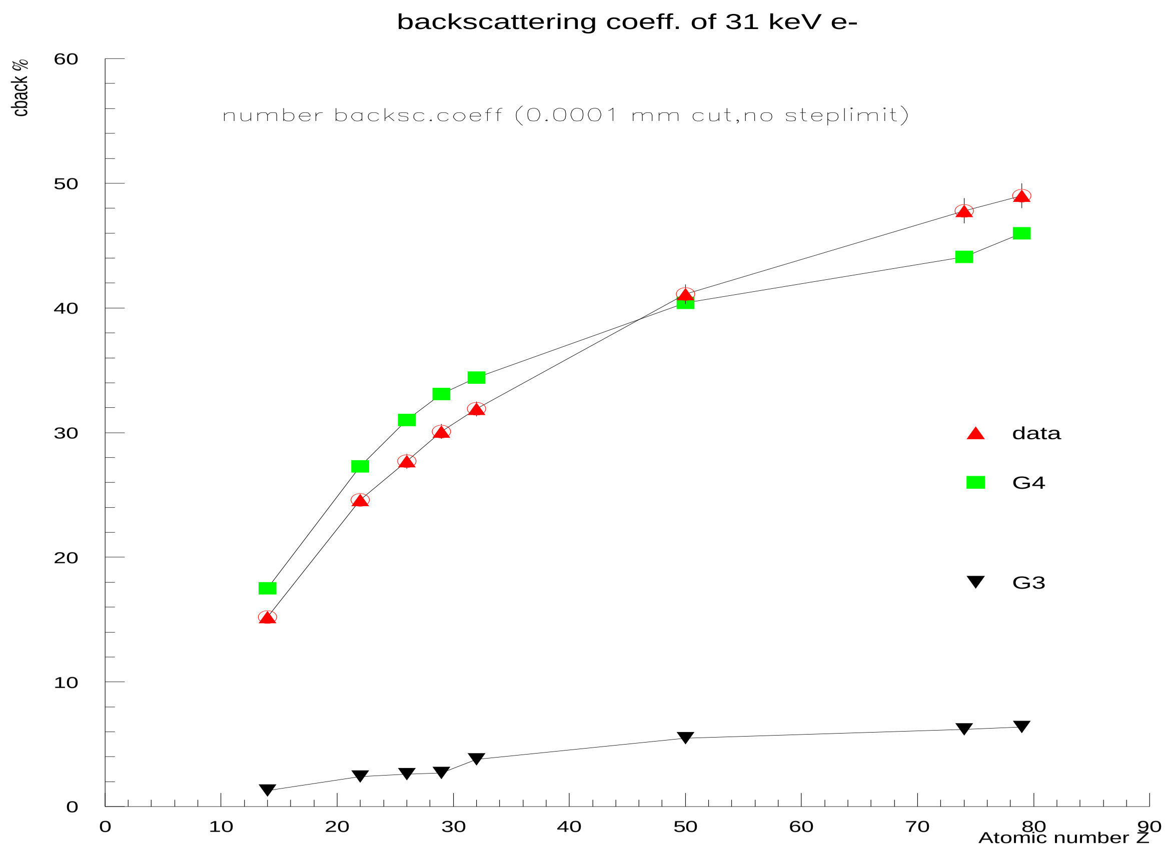
GEANT4 can do the job ! (some other simulation codes can simulate backscattering, but these are 'microscopic' or 'mixed' MC codes ...)

Some results follow ....

albedo : The incident beam is 10 electrons of 600 keV entering in 50 μm of Tungsten.

4 electrons are transmitted, 2 are backscattered.





backscattering of 31 keV e- from diff. targets(thick)

$\rightarrow$ G4 = data , G3 $\ll$ data

Čerenkov radiation

In a material with refractive index n, a charged particle emits photons if its velocity is greater than the local phase velocity of light.

The charged particle polarizes the atoms along its trajectory. These time dependent dipoles emit electromagnetic radiations.

If $v < c/n$ the dipole distribution is symmetric around the particle position, and the sum of all dipoles vanishes.

If $v > c/n$ the distribution is asymmetric and the total time dependent dipole is non nul, thus radiates.

mechanism of the Čerenkov radiation [Grupen96].

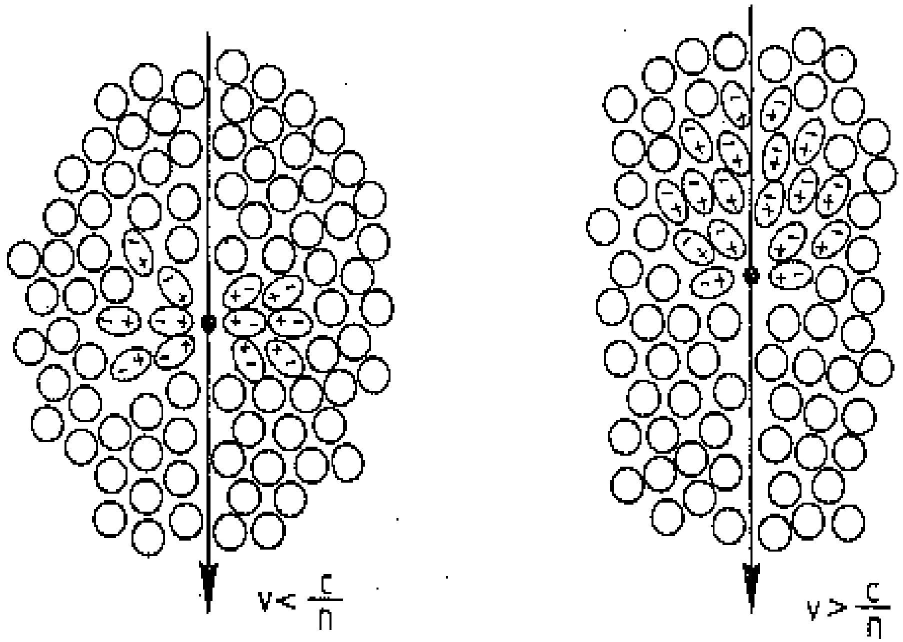
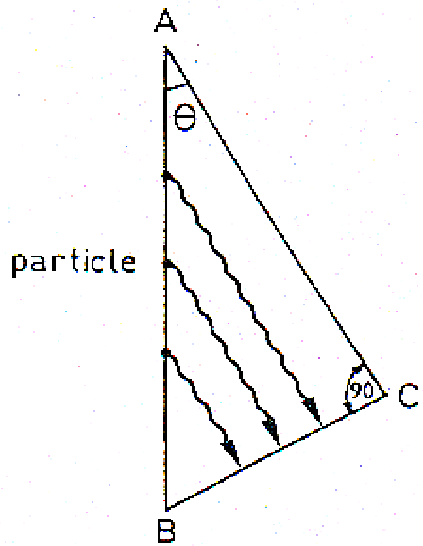


Fig. 6.7. Illustration of the Cherenkov effect [68].



The Huyghens construction gives immediately :

$$\cos\theta = \frac{1}{\beta n}$$

Thus :

$$\frac{1}{n} \leq \beta < 1 \Longrightarrow 0 \leq \theta < \arccos\frac{1}{n}$$

The number of photons produced per unit path length and per energy interval of the photons is

$$\frac{d^2N}{d\epsilon\, dx} = \frac{\alpha z^2}{\hbar c}\ \sin^2\theta = \frac{(\alpha z)^2}{r_e\, mc^2}\ \left[1 - \frac{1}{\beta^2\, n^2(\epsilon)}\right]$$

in which

$$\beta\, n(\epsilon) > 1$$

In the X-ray region $n(\epsilon) \approx 1$. There is no X-ray Čerenkov emission.

The average number of photons produced per unit path length :

$$\frac{dN}{dx} = \frac{(\alpha z)^2}{r_e\ mc^2} \int_{\epsilon_{min}}^{\epsilon_{max}} d\epsilon \left(1 - \frac{1}{\beta^2 n^2(\epsilon)}\right)$$

The number of photons produced per step is calculated from a Poissonian distribution with average value :

$$< n >= \text{StepLength}\ \frac{dN}{dx}$$

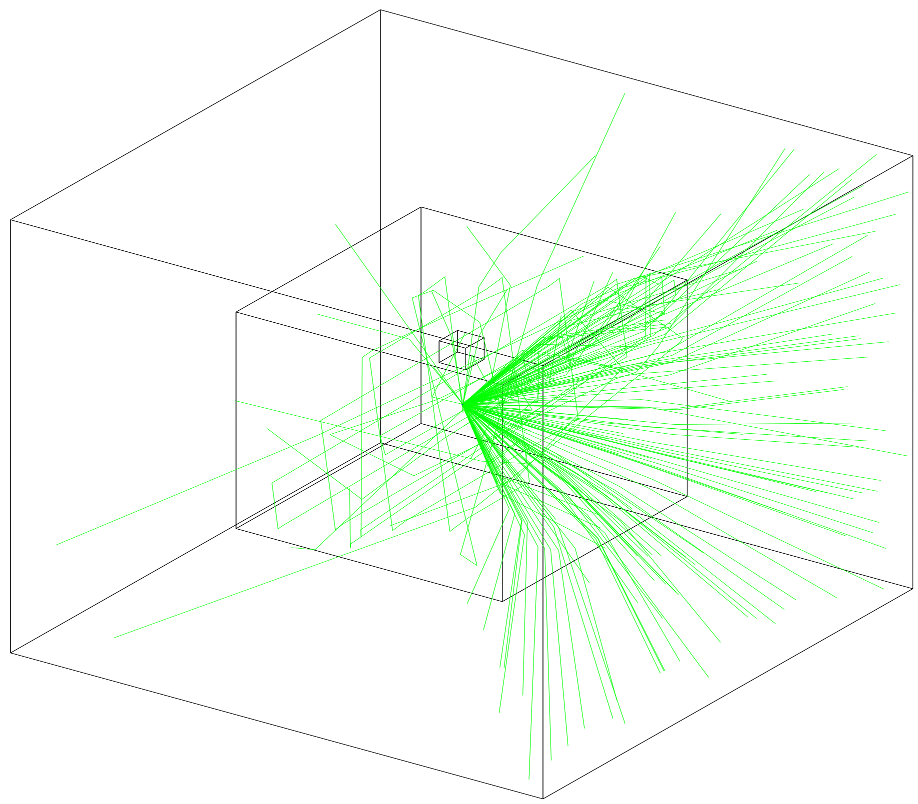
The generated photons are uniformly distribued along the track.

The energy distribution of the photon is sampled from the density function:

$$f(\epsilon) = \left[1 - \frac{1}{n^2(\epsilon)\beta^2}\right]$$

The Cerenkov radiation is an example of pure AlongStep process.

Cerenkov emission of optical photons by 15 MeV e^+ in water



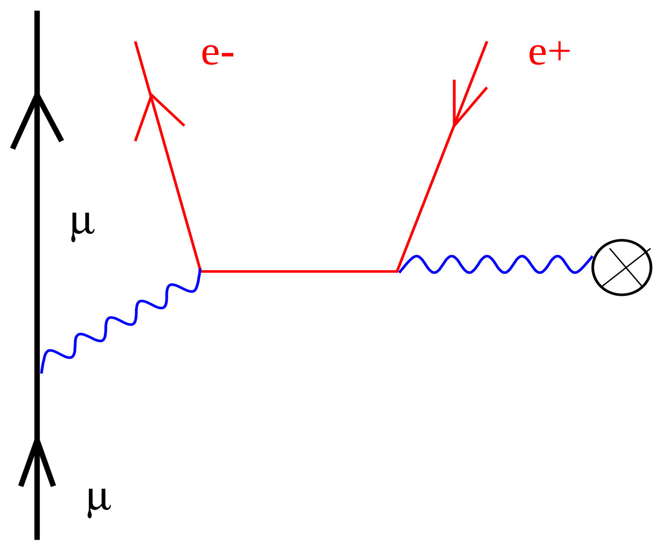
The energy lost by the charged particle due to Čerenkov emission is small compared to collision loss, even in gas :

$$\sim 10^{-1} \text{ to } 10^{-3} \text{ MeV}/(\text{g}/\text{cm}^2)$$

Direct (e^+, e^-) pair creation by muon

Creation of a (e^+, e^-) pair by virtual photon in the Coulomb field of the nucleus (for momentum conservation).

$$\mu + \text{nucleus} \longrightarrow \mu + e^+ + e^- + \text{nucleus}$$

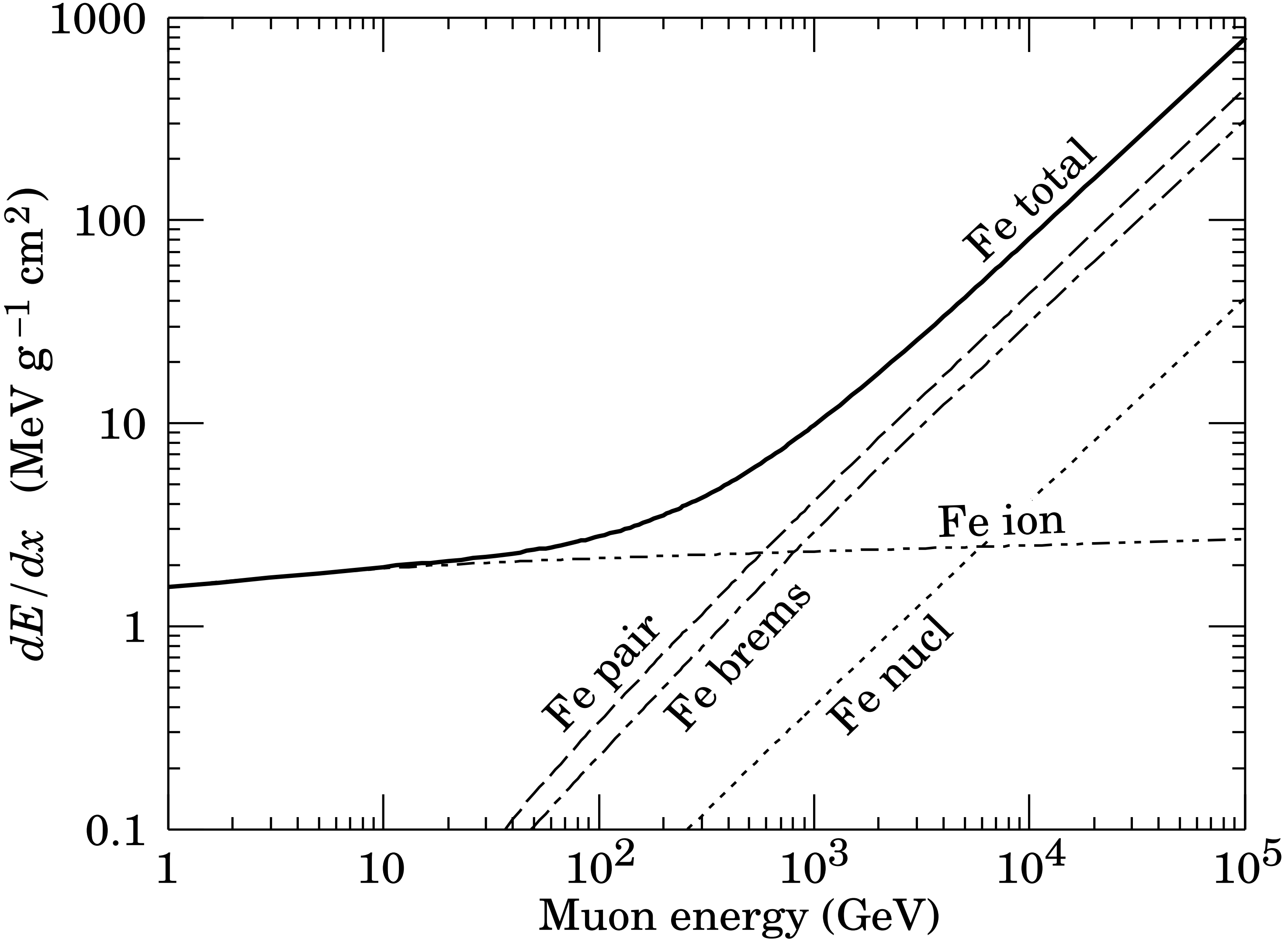


It is one of the most important processes of muon interaction.

At TeV muon energies, pair creation cross section exceeds those of other muon interaction processes in a wide region of energy transfers :

$$100 \text{ MeV} \leq \epsilon \leq 0.1 \; E_\mu$$

Average energy loss for pair production increases linearly with muon energy, and in TeV region this process contributes over 50 % to the total energy loss rate.


1000
100
10
1
0.1
dE/dx (MeV g^-1 cm^2)
1
10
10^2
10^3
10^4
10^5
Muon energy (GeV)
Fe total
Fe pair
Fe brems
Fe nucl
Fe ion

differential cross section

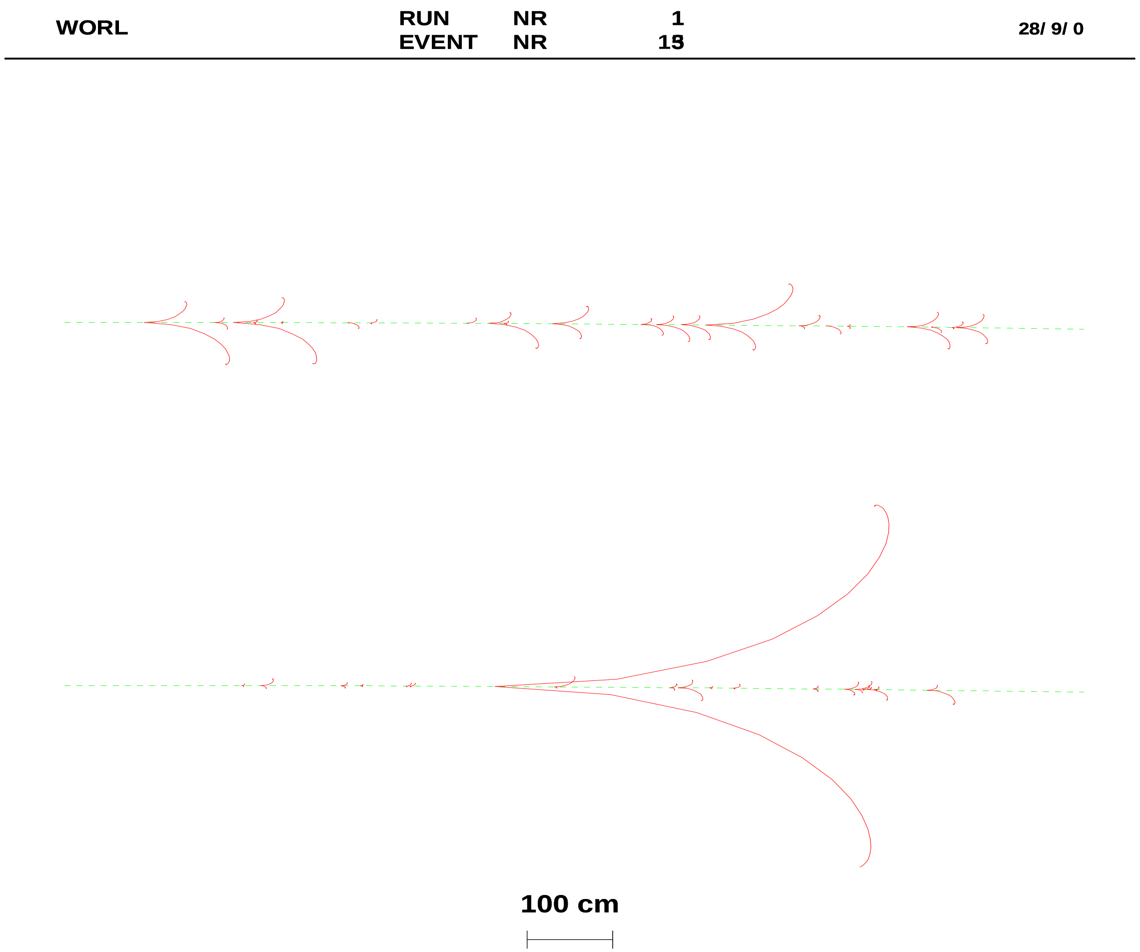
The differential cross section is given by Kokoulin et al. [Koko71].
It includes :

- screening of the field of the nucleus
- correction for finite nuclear size
- contribution from the atomic electrons [Keln97]
- ...

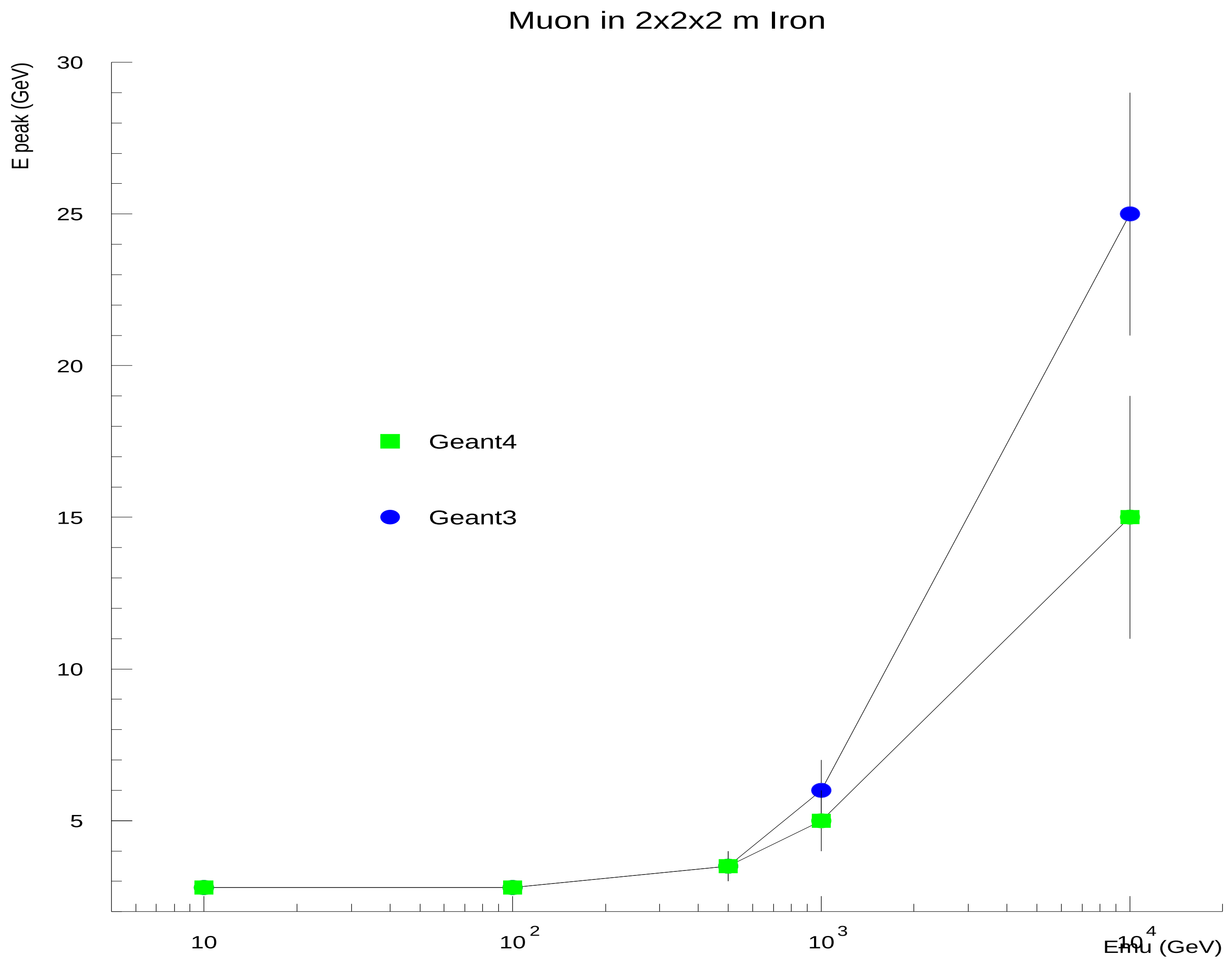
See [Koko71] for a complete discussion.

1 TeV muon in 10 meter of Fe (field 5 tesla).

direct pair creation only



energy deposited by high energy muons in a bloc of Iron



Energy-Range relation

Mean total pathlength of a charged particle of kinetic energy E :

$$R(E) = \int_{\epsilon=0}^{\epsilon=E} \left[\frac{d\epsilon}{dx}\right]^{-1} d\epsilon$$

In GEANT4 the energy-range relation is extensively used :

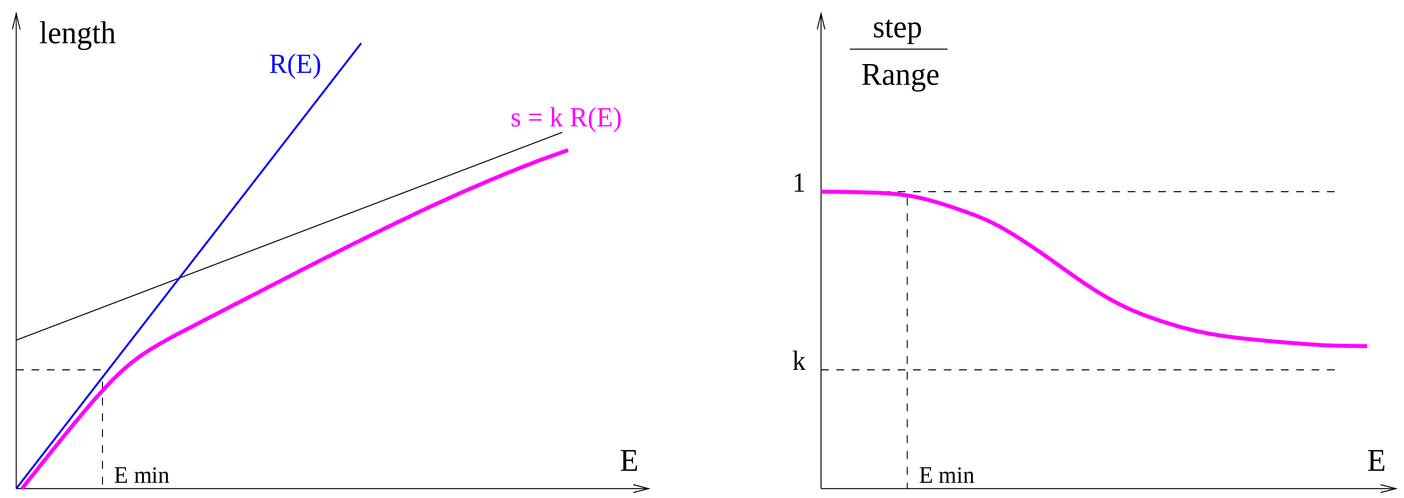
- to control the stepping of charged particles
- to compute the energy loss of charged particles
- to control the production of secondaries (cut in range)

control the stepping of charged particles

The continuous energy loss imposes a limit on the stepsize.

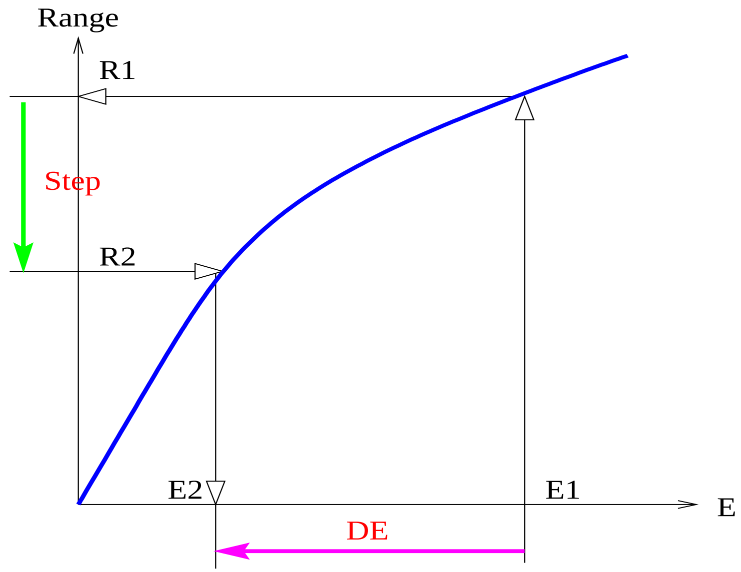
The cross sections depend of the energy. The step size must be small enough so that the energy difference along the step is a small fraction of the particle energy.

This constraint must be relaxed when $E \to 0$: the allowed step smoothly approaches the stopping range of the particle.



compute the mean energy loss of charged particles

The computation of the mean energy loss on a given step is done from the Range and inverse Range tables.



This is more accurate than $\Delta E = (dE/dx) * \text{stepLength}$.

On the same spirit, the time of life of the particle is updated from tables, automatically taking account that the particle velocity is slowing down along the step.

production thresholds (cuts) of secondaries

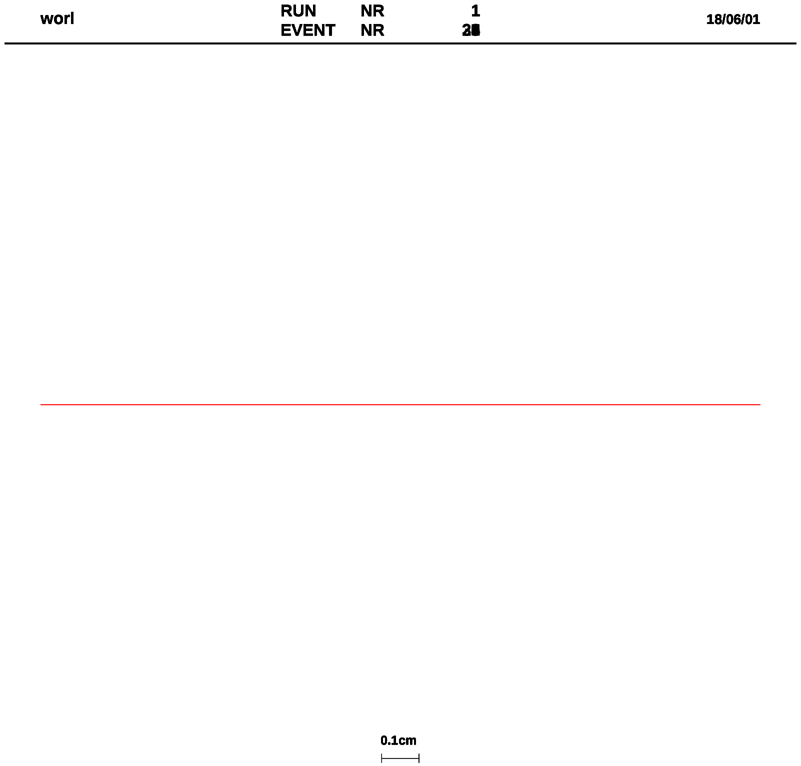
Production thresholds are expressed in range (instead in energy) for charged particles and photons (photon 'range' = abs.length)

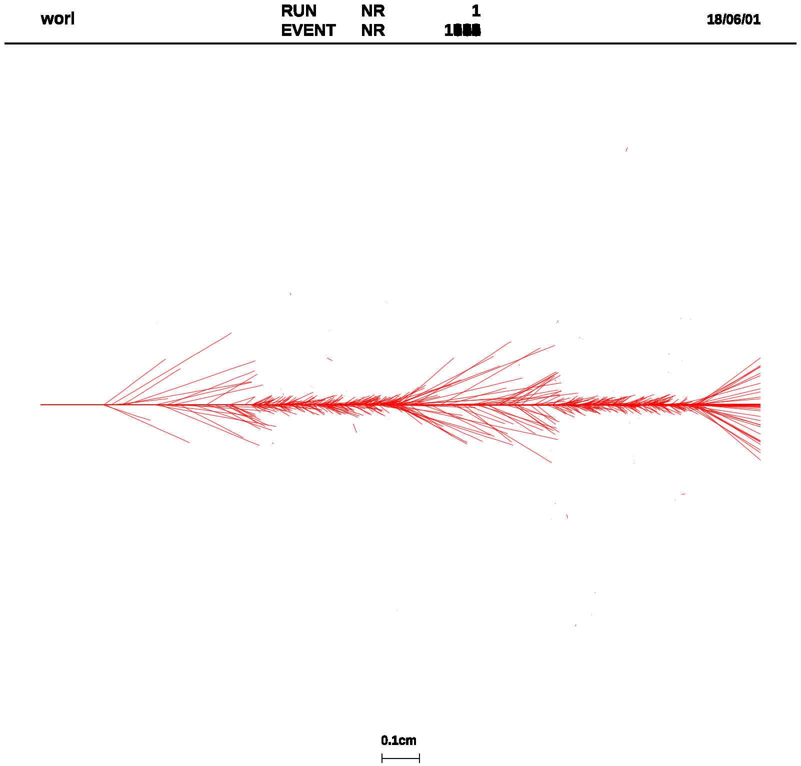
No difference in a homogenous material, but GEANT4 choice is better in general, e.g. sampling calorimeter.

example : Pb + liquidArgon + Pb + liquid Argon
each layer is few mm thick → cut/threshold can be 1(0.1) mm,
cuts in energy $E_{lAr}^{cut} < E_{Pb}^{cut}$ give 'coherent' physics
while using the same energy cut in both material gives 'not so good' physics in the case of high cut or degrades the efficiency (speed) for small cut value.

sampling: LAr(4mm) Pb(4mm). Protons 500 MeV. GEANT3

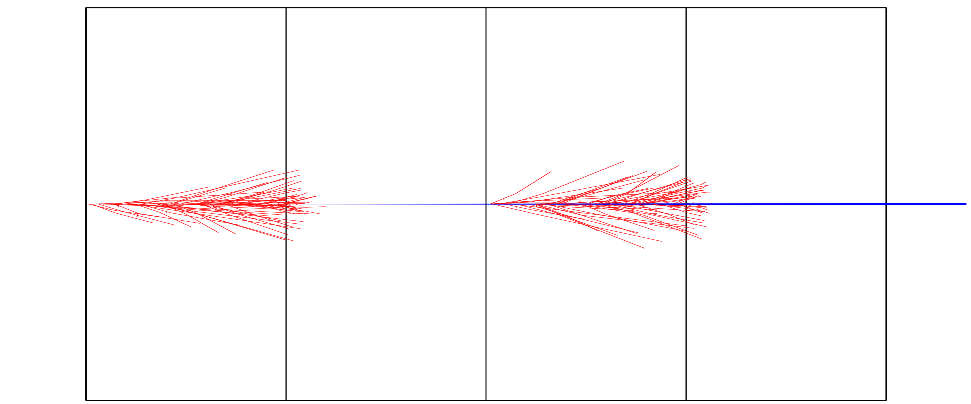
left: cut = 2MeV; right: cut = 450keV





Cut in range: sampling: LAr(4mm) Pb(4mm). Protons 500 MeV.

GEANT4 cut: 1.5mm =(450keV, 2MeV)

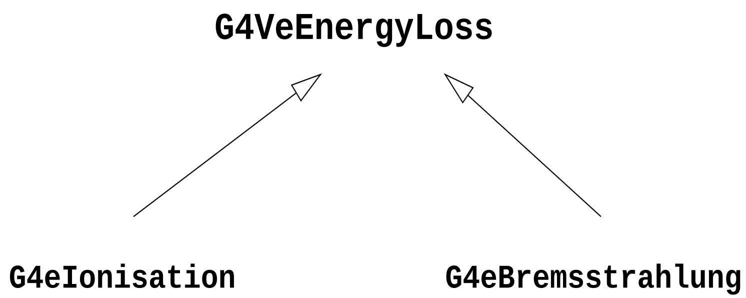


design details

Ionization and Bremsstrahlung cannot be independent :

$$\left[\frac{dE}{dx}\right]_{tot} = \left[\frac{dE}{dx}\right]_{ioni} + \left[\frac{dE}{dx}\right]_{brem}$$

$$R(E) = \int_{\epsilon=0}^{\epsilon=E} \left[\frac{d\epsilon}{dx}\right]_{tot}^{-1} d\epsilon$$



The processes compute the individual contributions.

The base class computes the sum and the range.

The base class is pure virtual : it cannot be directly instantiated.

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